Name: ________________________________

Teacher: ________________________________

Class: ________________________________

FORT STREET HIGH SCHOOL

2011
HIGHER SCHOOL CERTIFICATE COURSE
ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1
Time allowed: 2 hours
(plus 5 minutes reading time)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively.	1, 2	
Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry and polynomials.	3, 4	
Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as rates, kinematics and growth and decay.	5,6	
Synthesises mathematical solutions to harder problems such as projectiles and 3D trigonometry and communicates them in appropriate form.	7	

Question	1	2	3	4	5	6	7	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/84	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- **Each new question is to be started in a new booklet**

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

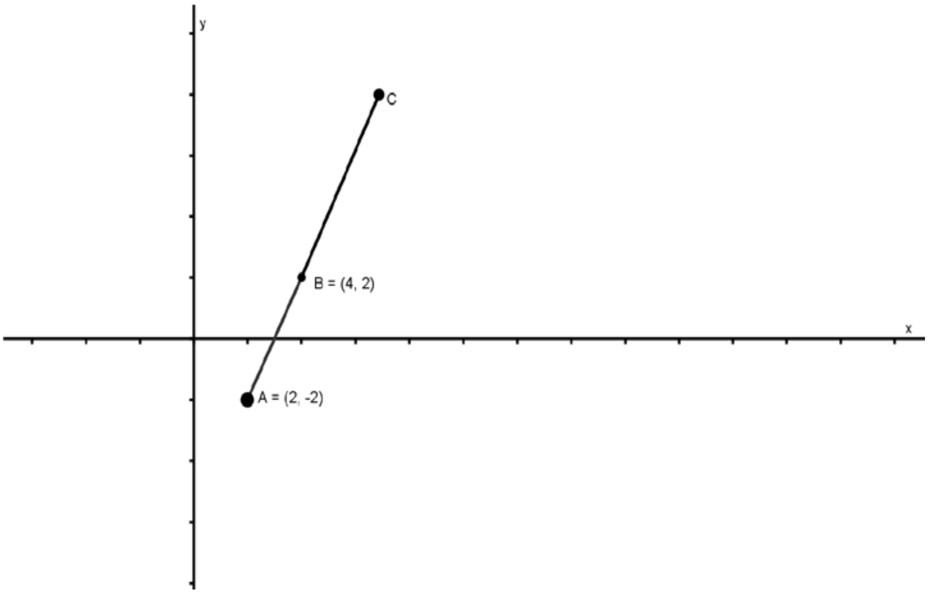
$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 (12 marks) **Use a SEPARATE writing booklet**

a) If A(2,-2) and B(4,2), find the co-ordinates of the point C (x, y), as shown in the diagram below, given that AC : CB = 7 : 5

2



b) Find the perpendicular distance from the point (1,2)

2

to the line $y = 3x - 5$

(Express the answer in exact rationalised form)

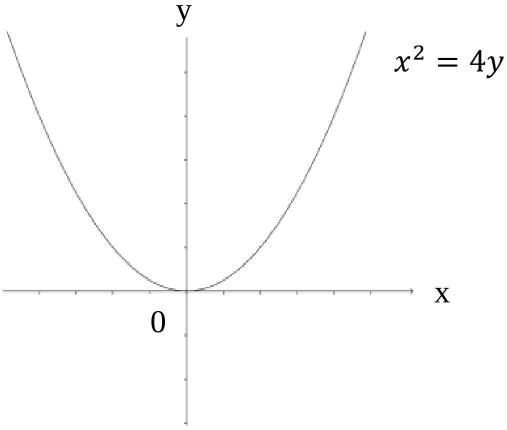
c) Solve $\frac{x+2}{x+1} \geq 3$ 3

d) Differentiate $e^{x} \tan^{-1}\frac{x}{2}$ 3

e) Evaluate $\lim_{x\to 0}\frac{2\sin 2x}{x}$ 2

Question 2 (12 marks) **Use a SEPARATE writing booklet**

a) T $(2t,t^2)$ is a point on the parabola $x^2 = 4y$ with focus F. The tangent to the parabola at T makes an acute angle θ with the line FT.



i) Show that the tangent to the parabola at T has gradient t. 1

ii) Find tan θ in simplest form in terms of t. 3

b) Evaluate $2\int_0^{\frac{\pi}{4}} cos^2\, 4x\, dx$ 3

c) Without using calculus, sketch $\boldsymbol{f(x)} = \frac{\mathbf{x^2}}{\mathbf{x^2 -4}}$ 3

Showing all the important features.

d) Consider the function $f(x) = x - e^{-2x}$.

Use one application of Newton's Method with an initial approximation of $x = 0.5$ to find the value of the x intercept on the graph of $y = f(x)$, giving the answer correct to two decimal places. 2

Question 3 (12 marks) **Use a SEPARATE writing booklet**

a) A, B, C and D are points on the circumference of a circle.
AB produced intersects DC produced at point P. AB =12cm, BP = 3cm and CD = 4cm.

i) Draw a clear sketch showing the above information. 1

ii) Find the length of CP. 2

b) The equation $8x^3 - 36\,x^2 + 22x + 21 = 0$ 5

has roots which form an arithmetic progression. Find the roots.

c) Find the area enclosed between the curves y = $\sin 2x$ and y = $2sin^2x.$

$0 \le x \le \frac{\pi}{4}.$ (Answer correct to 2 decimal places). 4

Question 4 (12 marks) **Use a SEPARATE writing booklet**

a) i) Express $\sqrt{3}\cos x$ - $\sin x$ in the form of $R\cos\ (x + a\ )$
where $0 < a < \frac{\pi}{2}$, and $R > 0$ 2

ii) Hence, solve $\sqrt{3}\cos x$ - $\sin x$ = $\sqrt{2}$ for $0 \le x \le \pi$ 2

(Answer in terms of π).

b) Show that $\frac{d}{dx}(\sin^{-1}x + \sqrt{1-x^2}\ ) = \sqrt{\frac{1-x}{1+x}}$,

hence evaluate $\int_0^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}}\,dx$ (Answer in exact form) 4

c) Use Mathematical Induction to prove the following result for positive integral values of n: 4

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{1}{1.3} + \frac{1}{3.5} + \cdots \ldots \ldots \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Question 5 (12 marks) **Use a SEPARATE writing booklet**

a) A particle P, initially at rest at $x = 2\ metres$ from the origin is moving along a straight line with an acceleration given by:

$$\frac{d^2x}{dt^2} = -4\left(x + \frac{16}{x^3}\right).$$

i) Show that if the velocity is v m/s at any given time, then 3

$$v^2 = \frac{64}{x^2} - 4x^2$$

ii) Hence, calculate the velocity when P is **halfway to** the origin. 1

iii) Calculate the **time** taken for the particle to reach the origin, given that

$$\frac{d}{dx}\left(\frac{1}{2}\cos^{-1}\left(\frac{x}{2}\right)^2\right) = \frac{-x}{\sqrt{16 - x^4}}$$

(Answer in terms of π). 4

b) Laura placed a cup of noodle soup with a temperature $95^{\circ}C$ in her room which has a temperature of $20^{\circ}C$. In 5 minutes the cup of noodle soup cools to $60^{\circ}C$. Assuming the rate of heat loss is proportional to the excess of its temperature above room temperature, that is:

$$\frac{dT}{dt} = -k(T - 20),$$

i) Show that $T = 20 + Ae^{-kt}$ is a solution of 1

$$\frac{dT}{dt} = -k(T - 20),$$

ii) If Laura likes to drink her noodle soup at $50°C$. Calculate the **extra** minutes she has to leave it to cool down.

(Answer to 1 decimal place). 3

Question 6 (12 marks) **Use a SEPARATE writing booklet**

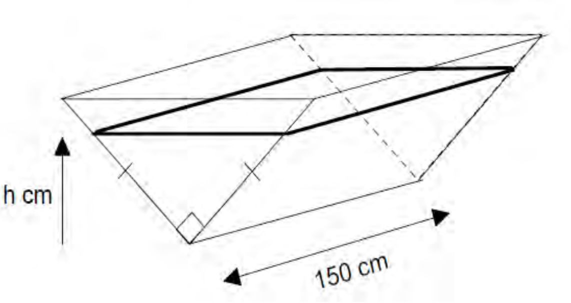
a) The displacement, $x\ cm$, of an object from the origin is given by
$x = 2sint - 3cost, \quad t \geq 0$, where time $t,$ is measured in seconds.

i) Show that the object is moving in Simple Harmonic Motion. 1

ii) At what time does the object **first** reach its maximum velocity? 2

(Answer correct to 2 decimal places).

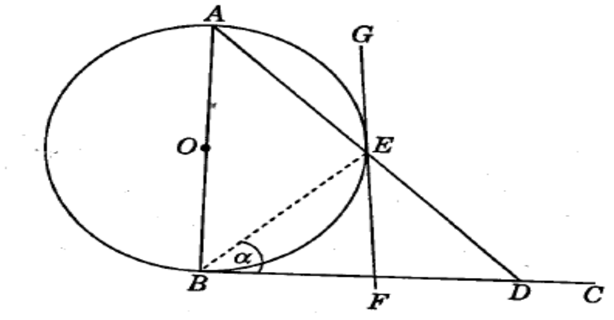
b) The diagram below shows a water trough 150cm long that has a cross section of a right angled isosceles triangle. Water is poured in at a constant rate of 3 litres per minute.



i) Show that when the depth of water is h cm, the volume of water in the tank is $150h^2\ cm^3$. 2

ii) Find the rate at which the water is rising when the depth is 5 cm. 3

c) In the diagram, AB is a diameter of the circle, centre **0**, and BC is a tangent to the circle at B. The line AED intersects the circle at E and BC at D. The tangent to the circle at E intersects BC at F, Let $< EBF = \alpha$.

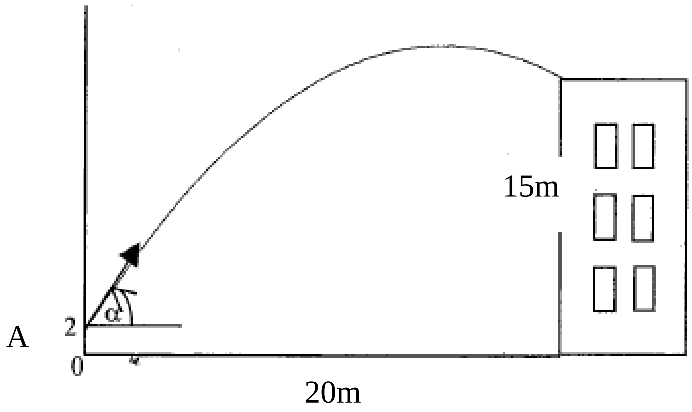


i) Copy the diagram into your Writing Booklet with all the relevant information. 1

ii) Prove that $< FED = \frac{\pi}{2} - \alpha$. 3

Question 7 (12 marks) **Use a SEPARATE writing booklet**

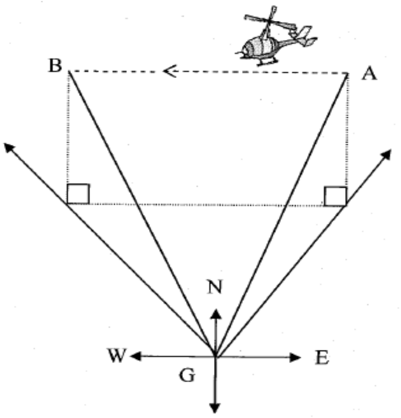
a) Andrew whose height is **2** metres throws a ball from area A to the roof of the Cohen building which is 15metres high. He throws the ball at an initial velocity of 25m/s, and he is 20 metres from the base of the building. (Assume $\ddot{x}$ **= 0** and $\ddot{y}$ = -10m/s^2)



i) Show that $y = x\ tan\alpha - \frac{5x^2}{v^2}$ (1+ $tan^2\alpha) + 2$, at any time t. 3

ii) Hence, find between which two angles of projection must he throw the ball to ensure that it lands on the roof of the building?
(Answer to the nearest degrees). 3

b) A helicopter flies due west from A to B at a constant speed of **420km/h**. From a point G on the ground the bearing of the helicopter when it is at A is $\boldsymbol{079^\circ T}$ with an angle of elevation $\boldsymbol{\beta}$. Four minutes later the helicopter is at B with a bearing from G being $\boldsymbol{302^\circ T}$ and an angle of elevation $\boldsymbol{32^\circ}$. The altitude of the helicopter is $\boldsymbol{h\ km}$.



i) Calculate the height of the plane to the nearest metre. 4

ii) Calculate the value of $\boldsymbol{\beta}$ to the nearest degree. 2

END OF PAPER

Extra

a) i) Show that $\frac{d}{dx}\log\,(cosec\,x+\cot x)=\,-cosec\,x$ 1

ii) Determine the volume generated when $y=-cosec\,x$, 2
is rotated about the $x-axis$, and the ordinates $x=\frac{\pi}{3}$ and $x=\frac{\pi}{2}$.
(Leave the answer in terms of π).

Solutions

1)

a) $A(2,-2)$ $B(4,2)$

$\times$

7 -3 C is external

Point C:

$x = \frac{7(4)-(5\times2)}{2}$ ✓ , $y = \frac{7(2)+10}{2}$

$\therefore (9, 12)$ ✓

① working
① point.

Many did not realise this was external division and still many errors in formula.

b) $d = \left|\frac{ax+by+c}{\sqrt{a^2+b^2}}\right|$ Point $(1,2)$
$a=3,\ b=-1,\ C=-5$

$= \left|\frac{3-2-5}{\sqrt{10}}\right|$ ✓

① working
① Answer

$= \frac{-4}{\sqrt{10}} \quad = \frac{4}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{2\sqrt{10}}{5}$ ✓

c) $\frac{x+2}{x+1} \geq 3$ $(x \neq -1)$
$(\times$ b.s by $(x+1)^2)$

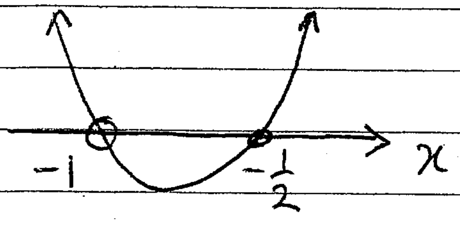
$(x+2)(x+1) \geq 3(x+1)^2$ ✓

$3(x+1)^2 - (x+1)(x+2) \leq 0$

$(x+1)[3x+3-(x+2)] \leq 0$

$(x+1)(2x+1) \leq 0$

$-1 < x \leq -\frac{1}{2}$ ✓ ✓



① working (any method)
① values
① correct Inequality signs.

$x = -1$ so need to take care not to put it in final answer.

e) $$\frac{d}{dx}\left(\overset{u}{e^{x}}\cdot \overset{v}{\tan^{-1}\frac{x}{2}}\right) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= e^{x}\cdot\frac{2}{4+x^2} + \tan^{-1}\frac{x}{2}\cdot e^{x} \quad ✓$$

$$✓ \quad = \frac{2e^{x}}{4+x^2} + e^{x}\tan^{-1}\frac{x}{2}$$

✓

$u = e^{x}$

$\frac{du}{dx} = e^{x}$

$V = \tan^{-1}\frac{x}{2}$

$\frac{dv}{dx} = \frac{2}{1+\left(\frac{x}{2}\right)^2}$

$= \frac{2}{4+x^2}$

① Product rule

① correct derivative of $\tan^{-1}\frac{x}{2}$

① Answer

f) Evaluate:

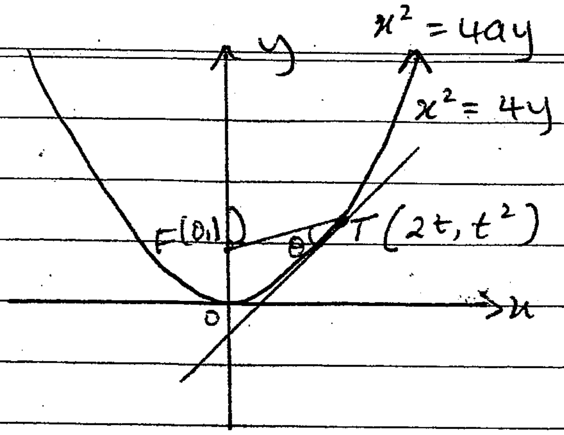
$$\lim_{x\to 0}\frac{2\sin 2x}{x}$$

$$= 4\lim_{x\to 0}\frac{\sin 2x}{2x} \quad ✓$$

$$= 4 \quad ✓$$

Question 2

a) i) $y = \frac{x^2}{4}$



$\frac{dy}{dx} = \frac{x}{2}$, $T(2t, t^2)$

$\frac{dy}{dx} = \frac{2t}{2}$

$= t$ ✓ — ①

$4a = 4$

$a = 1$

Some did not calculate the value of a.

ii) from ①

$m_T = t$

$m_{FT} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{t^2 - 1}{2t - 0}$

$= \frac{t^2 - 1}{2t}$ — ② ✓

$\tan\theta = \left| \frac{M_1 - M_2}{1 + M_1 M_2} \right|$

$= \left| \frac{(t^2+1)}{2t} \times \frac{2}{(t^2+1)} \right|$ ✓

$= \frac{1}{t}$ ✓

$M_1 - M_2$

$= t - \left(\frac{t^2 - 1}{2t}\right)$

$= \frac{2t^2 - t^2 + 1}{2t}$

$= \frac{t^2 + 1}{2t}$

$1 + M_1 M_2$

$= 1 + t \cdot \frac{t^2 - 1}{2t}$

$= 1 + \frac{t^2 - 1}{2}$

$= \frac{t^2 + 1}{2}$

many made mistakes in the algebraic manipulation of $\frac{M_1 - M_2}{1 + M_1 M_2}$

Some left the answer for $\tan\theta$ as an obtuse angle. (Did not read the question carefully)

b) $$2\int_{0}^{\frac{\pi}{4}} \cos^2 4x \, dx$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = \cos^2 x - (1 - \cos^2 x)$$
$$\cos 2x = 2\cos^2 x - 1$$
$$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$$

$$= 2\int_{0}^{\frac{\pi}{4}} \frac{1}{2}(\cos 8x + 1)\, dx$$

$$= \int_{0}^{\frac{\pi}{4}} (\cos 8x + 1)\, dx \quad ✓$$

$$= \left[\frac{1}{2}\sin 8x + x\right]_{0}^{\frac{\pi}{4}} \quad ✓$$

mostly well done.

$$= \left[\frac{\pi}{4} + \frac{1}{8}\sin\left(8 \times \frac{\pi}{4}\right)\right] - [0]$$

$$= \frac{\pi}{4} \quad ✓$$

c) $$f(x) = \frac{x^2}{x^2 - 4} \quad (x \neq \pm 2)$$

$$y = \frac{x^2}{(x-2)(x+2)}$$

$$\begin{array}{r|l} & \quad 1 \\ x^2 - 4 & \; x^2 \\ & -(x^2 - 4) \\ \hline & \quad\; 4 \end{array}$$

$$\therefore y = 1 + \frac{4}{x^2 - 4}$$

④

$$(y-1) = \frac{4}{(x-2)(x+2)}$$

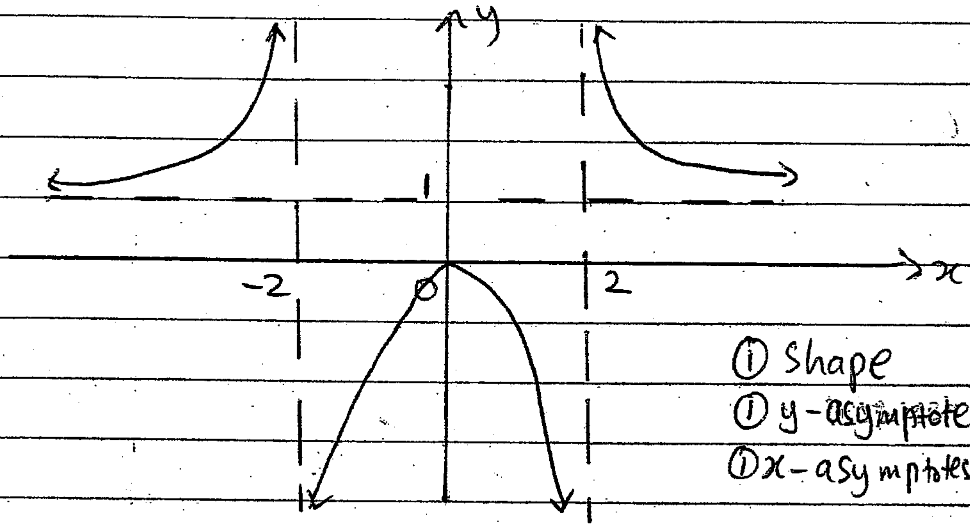
Intercepts:
$x=0,\ y=0$

Critical Points:

$$(x-2)(x+2)(y-1) = 4$$

$$x \neq \pm 2, \quad y \neq 1$$

many did not find the horizontal asymptote.



① Shape
① y-asymptote
① x-asymptotes

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d) $f(x) = x - e^{-2x} \Rightarrow f(0.5) = 0.5 - e^{-1}$
$= 0.132\ldots$

$f'(x) = 1+2e^{-2x}$
$f'(0.5) = 1+2e^{-1}$
$= 1.7358\ldots$

well done.

$$x = 0.5 - \frac{f(x)}{f'(x)}$$

$$= 0.5 - \frac{f(0.5)}{f'(0.5)} \checkmark$$

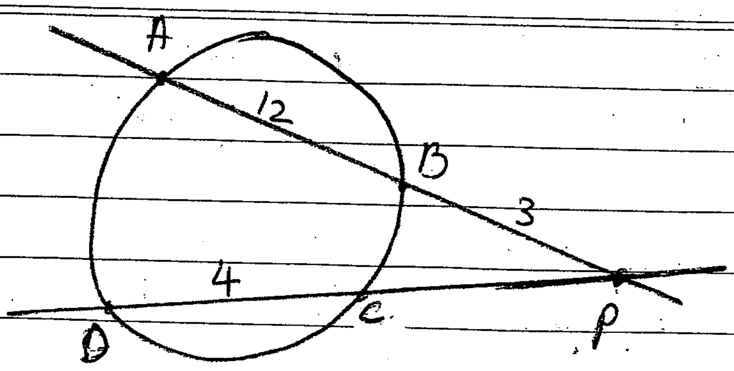
① Working
① Answer

$$= 0.5 - \frac{0.132\ldots}{1.7368\ldots}$$

$$= 0.42 \checkmark$$

Question 3

3)a i)



✓

ii) let $CP = x$

$x(x+4) = 3(15)$ ✓

$x^2 + 4x - 45 = 0$

$(x+9)(x-5) = 0$

$x = 5$ cm , $(x \neq -9)$

✓

Very poorly done students need to learn this rule. Also points are always cyclically named. DC - produced means to extend DC in that direction

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b) $8x^3 - 36x^2 + 22x + 21 = 0$

$\alpha + \beta + \gamma = \frac{36}{8} = \frac{9}{2}$ — ①

✓

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{22}{8} = \frac{11}{4}$ — ②

$\alpha\beta\gamma = \frac{-21}{8}$ — ③

✓

For AP : α, β, γ

$\boxed{\beta = \frac{\alpha+\gamma}{2}}$ or $\boxed{2\beta = \alpha + \gamma}$ — ④

Many made a reasonably simple question very complicated & too many simple arithmetic errors.

Sub ④ into ①

$3\beta = \frac{9}{2}$

$\beta = \frac{3}{2}$ ✓

∴ from ①

$\alpha + \gamma = 3$

$\alpha = 3 - \gamma$

from ③

$\alpha\gamma \cdot \frac{3}{2} = -\frac{21}{8}$

$\alpha\gamma = -\frac{7}{4}$ — ⑤

∴ $\alpha(3-\alpha) = \frac{-7}{4}$ ✓

$\begin{matrix} 2\alpha & & 1 \\ & \times & \\ 2\alpha & & -7 \end{matrix}$

$4\alpha^2 - 12\alpha = 7$

$4\alpha^2 - 12\alpha - 7 = 0$

$(2\alpha + 1)(2\alpha - 7) = 0$

$\alpha = -\frac{1}{2}, \frac{7}{2}$ ✓

$\gamma = 3 - \alpha$
$= \frac{7}{2}$

∴ The roots are ; $-\frac{1}{2}, \frac{3}{2}, \frac{7}{2}$

difference = 2

② Working (any method)
① 1 root solved
① working to find the other roots.
① Answers.

c) $\displaystyle\int_0^{\frac{\pi}{4}} \sin 2x - 2\sin^2 x \, dx$ | $2\sin^2 x = 1 - \cos 2x$

$= \displaystyle\int_0^{\frac{\pi}{4}} (\sin 2x + \cos 2x - 1)\, dx$ ✓

$= \left[-\frac{\cos 2x}{2} + \frac{\sin 2x}{2} - x\right]_0^{\frac{\pi}{4}}$ ✓

$= \left[-0 + \frac{1}{2} - \frac{\pi}{4}\right] - (-1 + 0 - 0)$ ✓

$= 0.21 \text{ cm}^2$ ✓

① Substitution
① Integration
① Values
① Answer

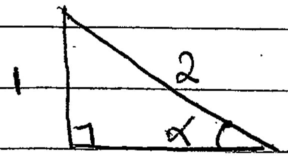
Need to always take absolute value in case wrong curve on top. Also question asked to evaluate to 2 dec places.

⑦

Question 4

a) i) $\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$

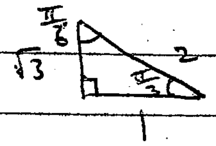
mostly well executed



$$\tan\alpha = \frac{b}{a}$$

$$= \frac{1}{\sqrt{3}}$$

$$= \frac{\pi}{6}$$

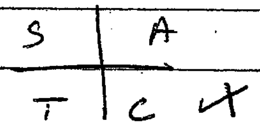


$$\therefore \sqrt{3}\cos x - \sin x = \overset{\checkmark}{2}\cos\left(x + \overset{\checkmark}{\frac{\pi}{6}}\right)$$

① The amplitude
① The acute angle

ii) hence:

$$2\cos\left(x + \frac{\pi}{6}\right) = \sqrt{2}$$



$$\cos \quad x + \frac{\pi}{6} = \frac{\sqrt{2}}{2} \qquad (0 \le x \le \pi)$$

$$x + \frac{\pi}{6} = \frac{\pi}{4}\checkmark, \quad \frac{7\pi}{4}$$

$$x = \frac{\pi}{12}, \quad \frac{19\pi}{12}$$

✓

$x = \frac{\pi}{12}$ is the only sol$^{\underline{n}}$.

Full marks for answer of $\frac{\pi}{12}$ without showing elimination of $\frac{19\pi}{12}$

8

b) $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$

straight forward for most students, some making numerical errors.

$= \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$ ✓

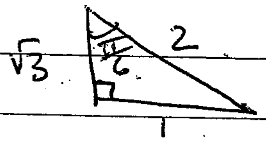
$= \frac{1-x}{\sqrt{1-x^2}}$

$= \frac{(1-x)}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}$

$= \frac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}$ ✓

$= \sqrt{\frac{(1-x)}{(1+x)}}$

$\therefore \int_0^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}}\, dx$



$= \left[\sin^{-1}x + \sqrt{1-x^2}\right]_0^{\frac{1}{2}}$

✓

$= \left[\frac{\pi}{6} + \sqrt{\frac{3}{4}}\right] - \left[0+1\right]$

$= \frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1$ ✓

⑨

c) $S(n) = \sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \dots + \frac{1}{(2n-1)(2n+1)}$

$= \frac{n}{2n+1}$

Step 1:

let $n=1$

$LHS = \frac{1}{(1)(3)}$ $\quad RHS = \frac{1}{3}$

$\therefore$ LHS = RHS

$\therefore S(1)$ is true

Step 2:

Assume $S(k)$ is also true ✓

$\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$ — ①

Step 3:

Prove $S(k+1)$

LHS : from ①

$\frac{k}{2k+1} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$

$\frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$

RHS :

$= \frac{k+1}{2(k+1)+1}$

$= \frac{k+1}{2k+3}$

✓

errors included assumption containing $\frac{1}{(2k-1)(2k+1)}$ on LHS only, ie not whole sum, working on bs in proof, insufficent conclusion (not linking the previous steps)

$LHS = \frac{k(2k+3)+1}{(2k+1)(2k+3)}$

X 1 1

$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$ ✓

$= \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)} = \frac{k+1}{2k+3}$

$= RHS$

① assumption

① using the assumption to prove for k+1

① Proving

① Conclusion

$\therefore$ Since it is true for $n=1$, $n=k$, $n=k+1$ $(n=1, n=2, n=$

then it is true for $n \geq 1$ ✓

⑩

Question 5

a) i) $\dfrac{d^2x}{dt^2} = -4\left(x + \dfrac{16}{x^3}\right)$

$\dfrac{d}{dx}\left(\dfrac{v^2}{2}\right) = -4\left(x + 16x^{-3}\right)$

$\dfrac{v^2}{2} = -4\int x + 16x^{-3}\,dx$ ✓

$\dfrac{v^2}{2} = -4\left(\dfrac{x^2}{2} + \dfrac{16x^{-2}}{-2}\right) + c$

$\dfrac{v^2}{2} = -2x^2 + 64x^{-2} + c$ ✓

$v^2 = \dfrac{64}{x^2} - 4x^2 + c$

Initially at rest, $\therefore c = 0$ ✓
ie : $v = 0,\ x = 2$

$v^2 = \dfrac{64}{x^2} - 4x^2$ as required.

Some tried to integrate (cannot integra x with respec to t)!!

— some ignored 'c' + hence di not evaluate it

ii) $v^2 = \dfrac{64}{x^2} - 4x^2 \quad (x = 1)$

$v^2 = 64 - 4$
$v^2 = 60$
$v = -2\sqrt{15}$ m/s (-7.75 m/s)

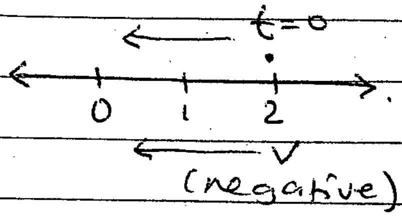
* negative velocity ✓

Many did not realise velecity was negative (moving in a -ve directio

No mark awarded

iii) $V = -\left[\dfrac{64 - 4x^4}{x^2}\right]^{\frac{1}{2}}$

$$\frac{dx}{dt} = \left[\frac{4(16 - x^4}{x^2}\right]^{\frac{1}{2}}$$



$$= -2\left(\frac{16 - x^4}{x^2}\right)^{\frac{1}{2}}$$

$$\therefore \frac{dt}{dx} = -\frac{x}{2\sqrt{16 - x^4}} \checkmark$$

— negative sign was omitted often.

$$t = \frac{1}{2}\int \frac{-x}{\sqrt{16 - x^4}}\,dx \checkmark$$

some used 'c' & used t=0, x=2 to evaluate it

$$= \frac{1}{2}\left[\frac{1}{2}\cos^{-1}\left(\frac{x}{2}\right)^2\right]_2^0 \checkmark$$

← Many had trouble using the integral given correctly.

$$= \frac{1}{4}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{8} \text{ seconds.} \checkmark$$

$\therefore$ It takes $\frac{\pi}{8}$ s to get to the origin.

① expression for $\dfrac{dt}{dx}$

① expression for t

① Integration. & finding c

① Answer

⑫

b) i) $T = 20 + Ae^{-Kt}$ $(Ae^{-Kt} = T - 20)$

→ need to state this. If it is a show question, you can't just write down what is req'd. (No longer need to integrate to show T)

$$\frac{dT}{dt} = -KAe^{-Kt} \quad ✓$$

$$\downarrow$$

$$= -K(T-20)$$

∴ It is the solution.

ii) $t = 0$

$95 = 20 + Ae^{-0}$

$\therefore A = 75$

Some did not find 'A' first which caused further errors

$T = 20 + 75e^{-Kt}$

$t = 5$ $\quad 60 = 20 + 75e^{-5K}$

$75e^{-5K} = 40$

$-5K = \ln\left(\frac{8}{15}\right)$

$-K = \frac{1}{5}\ln\left(\frac{8}{15}\right)$

① expression/value for A/k

Method executed quite well

or k=

$T = 50°$.

① Value for t.

① Extra time

Some did not give the extra time.

$75e^{-Kt} = 30$

$\frac{1}{5}\ln\left(\frac{8}{15}\right)t = \ln\left(\frac{2}{5}\right)$

$t = \ln\left(\frac{2}{5}\right) \div \frac{1}{5}\ln\left(\frac{8}{15}\right)$.

$= 7.3$ minutes

(Some had answers under 5 minutes ⇒ so should have realised there was an error)

∴ Extra time = 2.3 mins ✓

13

Accuracy of answer 1 dp!!

Question 6

a) i) $x = 2\sin t - 3\cos t$

$\dot{x} = 2\cos t + 3\sin t$

$\ddot{x} = -2\sin t + 3\cos t$

$= -(2\sin t - 3\cos t)$ — ①

$\ddot{x} = -x$ ✓ $(n = 1)$

∴ motion is in SHM.

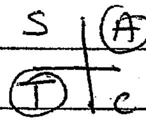
Need to demonstrate $\ddot{x} = -n^2 x$ not just rewrite x.

ii) · at maximum velocity:

$a = \ddot{x} = \frac{dv}{dt} = 0$

from ①

$3\cos t = 2\sin t$



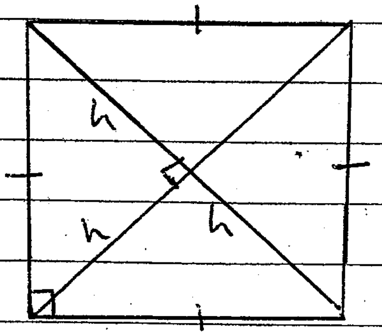
$\cancel{\tan t} = \frac{3}{2}$ ✓

$t = 0.98$ sec. ✓

~~Maximum~~ ~~changing~~ ~~[illegible]~~ ~~[illegible]~~

Need to have calculator in radian measure.

b) i)



Area $= \frac{1}{2}(h \times 2h)$ ✓

$= h^2$

$\therefore V = AH$ ✓

$= h^2(150)$

$= 150h^2$ cm³

All units must be same so 3L needs to be converted to cm³.

ii) $\frac{dV}{dt} = 3\,l/\text{min}$, $\frac{dh}{dt} = ?$, $h = 5$ cm

$\frac{dV}{dt} = \frac{dV}{dh}\boxed{\frac{dh}{dt}} = ?$

$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$ ✓

$= 3000 \div 300h$ ✓

$= \frac{30}{(3 \times 5)}$ ✓

$= 2$ cm/min

$\therefore \frac{dh}{dt} = 2$ cm/min

① expression for $\frac{dh}{dt}$

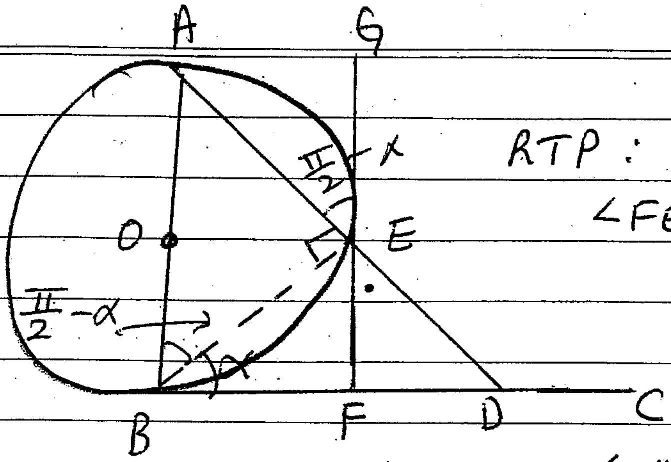
① working

① Answer

⑭

c)

i)



RTP :

$\angle FED = \frac{\pi}{2} - \alpha$

✓ marked on the diagram

ii) $\angle AEB = \frac{\pi}{2}$ (AB is a diameter)

$\angle ABF = \frac{\pi}{2}$ (tangent ⊥ diameter)

$\therefore \angle ABE = \frac{\pi}{2} - \alpha$ ✓

✓

$\angle ABE = \angle AEG$ (∠ in alt. segment)

$= \frac{\pi}{2} - \alpha$

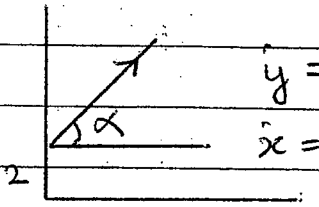
✓

$\angle AEG = \angle DEF$ (vertically opp)

$= \frac{\pi}{2} - \alpha$

Question 7

a) i) $\ddot{y} = -10$



$\dot{y} = 25\sin\alpha$

$\dot{x} = 25\cos\alpha$

$\dot{y} = -\int 10\,dt$

$\dot{y} = -10t + c$, $t=0$, $\dot{y} = 25\sin\alpha$

$\therefore \dot{y} = 25\sin\alpha - 10t$

OR $\dot{y} = V\sin\alpha - 10t$

$y = \int 25\sin\alpha - 10t \, dt$

$= 25t\sin\alpha - 5t^2 + c$ ✓

$t = 0$, $y = 2$ $\therefore c = 2$

$\therefore \boxed{y = 25t\sin\alpha - 5t^2 + 2}$

OR

$\boxed{y = Vt\sin\alpha - 5t^2 + 2}$ — ①

Horizontal:

$x = Vt\cos\alpha$

$\therefore t = \dfrac{x}{V\cos\alpha}$ — ②

Sub ② into ①

✓

$$y = V\left(\frac{x}{V\cos\alpha}\right)\sin\alpha - 5\left(\frac{x^2}{V^2\cos^2\alpha}\right) + 2$$

$$y = x\tan\alpha - \frac{5x^2}{V^2\cos^2\alpha} ✓ + 2$$

$$\therefore y = x\tan\alpha - \frac{5x^2}{V}(1+\tan^2\alpha) + 2$$

Many students ~~St~~ did not derive equations of motion.

① Integration of $y, x, \dot{y}$ and $\dot{x}$

① Substitution

① expression for y.

$\frac{1}{\cos^2\alpha} = \sec^2\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$

$\tan^2\alpha + 1 = \sec^2$

ii) $y = x\tan\alpha - \frac{5x^2}{V^2}(1+\tan^2\alpha)+2$

$15 = 20\tan\alpha - \frac{16}{5}(1+\tan^2\alpha)+2$ ✓

$75 = 100\tan\alpha - 16 - 16\tan^2\alpha + 10$

$16\tan^2\alpha - 100\tan\alpha + 81 = 0$ ✓

$\tan\alpha = \frac{100 \pm \sqrt{100^2-4(16)(81)}}{32}$

$= \frac{100 \pm 69.40}{32}$

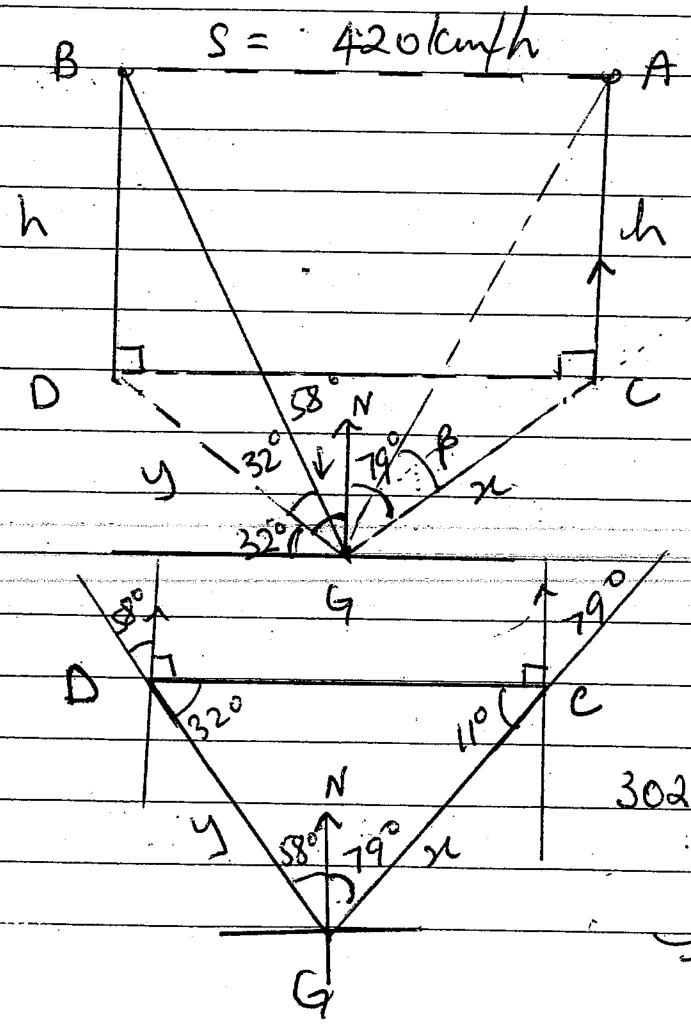
$\alpha = 44^\circ$ and 79°

$\therefore 44^\circ \le \alpha \le 79^\circ$ ✓

More care needed to be taken with these steps.

① Substitution
① quadratic formula
① Answer.

b) i)

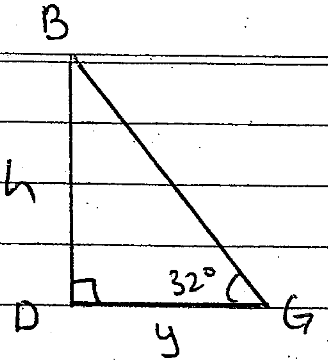


let CG = x
DG = y

many students did not draw diagrams or confused bearings with oblique angles.

302 − 270 = 32°

17



$\tan 32° = \frac{h}{y}$

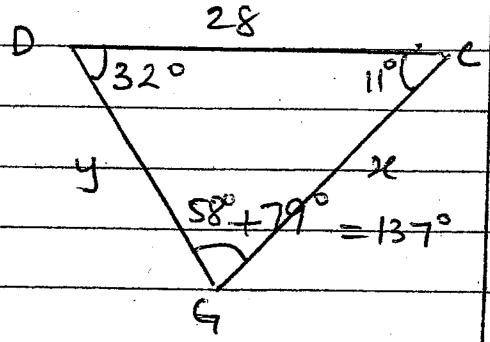
$y = \frac{h}{\tan 32°}$ —— ①

✓

DC = speed x time

$= 420 \times \frac{4}{60}$

$= 28$ km



$\frac{y}{\sin 11°} = \frac{28}{\sin 137°}$ ✓

$y = \frac{28 \sin 11°}{\sin 137°}$ —— ②

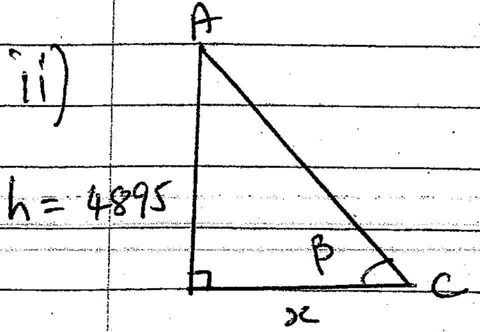
Sub ① into ②

$\frac{h}{\tan 32°} = \frac{28 \sin 11°}{\sin 137°}$ ✓

$h = \tan 32° \times \frac{28 \sin 11°}{\sin 137°}$

$= 4.895$ km ✓ (4895 m)

ii)



from △ CDG

$\frac{x}{\sin 32°} = \frac{28000}{\sin 137°}$

$x = 21756$ ✓

$\therefore \tan \beta = \frac{4895}{21756}$

$\beta = 13°$ ✓

18

END